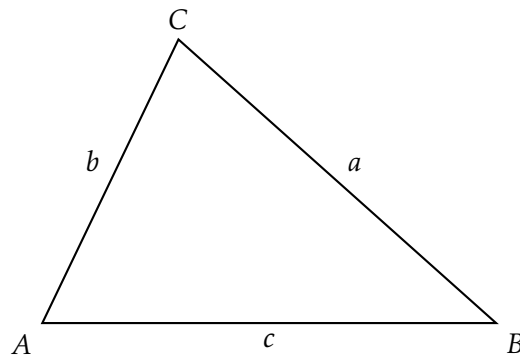


The Sine Rule

Things you should already know

- SOHCAHTOA for right-angled triangles
- Solving trig equations over $0^\circ \leq \theta \leq 360^\circ$
- Angles in a triangle sum to 180°

Labelling Convention



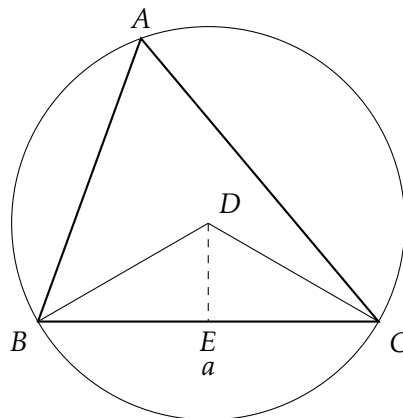
Capital letters for angles, lowercase for the side **opposite** that angle. So side a is opposite angle A .

Fact — The Sine Rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use this when you have a **matching pair** — a side and the angle opposite it.

Proof



Example

In $\triangle ABC$, $a = 7$ cm, $A = 58^\circ$, $B = 23^\circ$. Find the length b .

Example

In $\triangle ABC$, $a = 7$ cm, $A = 128^\circ$, $b = 6$ cm. Find angle B .

Textbook Exercises: SPSC: 4.10 — Exercise 3A, questions 1–10.

Dr Frost: Sine and Cosine Rules slides — sine rule section.

Corbett Maths: Sine Rule practice questions (Video 314).

Maths Genie: Sine Rule worksheet.

Maths4Everyone: Sine Rule booklet.

The Cosine Rule

Example

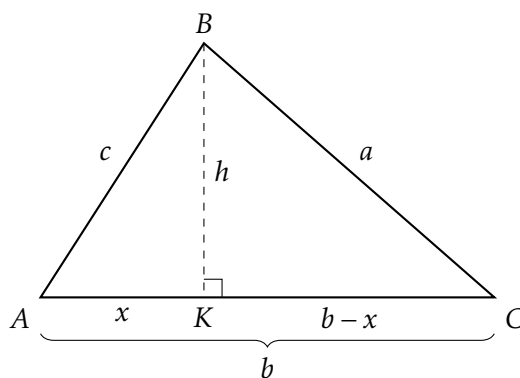
In $\triangle ABC$, $b = 8$ cm, $c = 5$ cm, $A = 60^\circ$. Can you use the sine rule? Why or why not?

Fact — The Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Use this when you have **two sides and the included angle (SAS)**, or **all three sides (SSS)**.

Proof



Finding a side (SAS)**Example**

In $\triangle ABC$, $b = 9$ cm, $a = 8$ cm, $C = 127^\circ$. Find c .

Example

In $\triangle PQR$, $p = 11$ cm, $r = 7$ cm, $Q = 48^\circ$. Find q .

Finding an angle (SSS)**Example**

In $\triangle ABC$, $a = 8$, $b = 9$, $c = 10$. Find the largest angle.

Example

In $\triangle ABC$, $a = 5$, $b = 6$, $c = 9$. Find angle C .

Textbook Exercises: SPSC: 4.10 — Exercise 3A, questions 11–15.

Dr Frost: Sine and Cosine Rules slides — cosine rule section.

Corbett Maths: Cosine Rule practice questions (Video 81).

Maths Genie: Cosine Rule worksheet.

Maths4Everyone: Cosine Rule booklet.

Area of a Triangle: $\frac{1}{2}ab \sin C$ **Example**

You know two sides and the included angle of a triangle. Can you find its area without finding the height first?

Fact — Area of a triangle:

$$\text{Area} = \frac{1}{2}ab \sin C$$

where a and b are two sides and C is the **included angle** between them.

Proof**Example**

Find the area of $\triangle ABC$ where $a = 7$ cm, $b = 8.5$ cm, $C = 72^\circ$.

Example

Find the area of $\triangle ABC$ where $a = 12.5$ cm, $A = 48^\circ$, $B = 65^\circ$.

Choosing the Right Rule

Fact — Decision flowchart:

1. Do you have a **matching pair** (side + opposite angle)?
 - Yes → **Sine Rule**
2. Do you have **two sides + included angle** (SAS)?
 - Yes → **Cosine Rule** (to find the third side)
3. Do you have **three sides** (SSS)?
 - Yes → **Cosine Rule** (to find an angle)
4. Always draw a diagram!
5. You may need to use $A + B + C = 180^\circ$.

Textbook Exercises: SPSC: 4.10 — Exercise 3A, questions 16–29.

Dr Frost: Sine and Cosine Rules slides — area of a triangle and mixed problems.

Corbett Maths: Area of a Triangle using Sine practice questions (Video 23b).

Maths Genie: Area of a Triangle worksheet (non-right-angled).

Maths4Everyone: Area of a Triangle (Trig) booklet.

The Ambiguous Case of the Sine Rule

Example

In $\triangle ABC$, $a = 10$ cm, $b = 7$ cm, $A = 40^\circ$. Find angle B .

Example

In $\triangle ABC$, $a = 5$ cm, $b = 8$ cm, $A = 30^\circ$. Find angle B .

Fact — The ambiguous case occurs when using the sine rule to find an angle. Since $\sin \theta = \sin(180^\circ - \theta)$,

there may be **two valid solutions**. Always check whether the second solution gives a valid triangle (angles summing to less than 180°).